On brick finiteness of finite-dimensional algebras

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Outline

Introduction

Goal of Algebraic Representation Theory

Classify all indecomposable modules of a given algebra A and all morphisms between them, up to isomorphism.

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- **rep-finite** if the number of indecomposable modules is finite.
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- **wild** if there exists a faithful exact K-linear functor from the module category of $K\langle x, y\rangle$ to $\operatorname{mod} A$.

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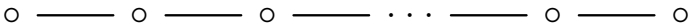
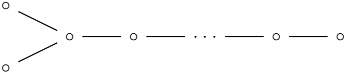
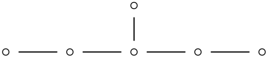
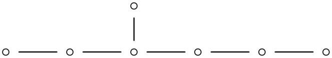
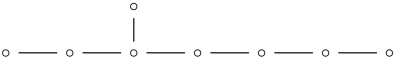
Quiver Representation Theory

Any (basic, connected) algebra A over an algebraically closed field K is isomorphic to a **bound quiver algebra** KQ/I.

Rep-finite path algebra

Gabriel's Theorem (Gabriel, 1972)

A path algebra $A = KQ$ is rep-finite if and only if the underlying graph of Q is one of Dynkin graphs:

- $\mathbb{A}_n$: 
- $\mathbb{D}_n$: 
- $\mathbb{E}_6$: 
- $\mathbb{E}_7$: 
- $\mathbb{E}_8$: 

Trichotomy Theorem (Drozd, 1977)

The representation type of an algebra A (over K) is exactly one of rep-finite, tame and wild.

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It leads to two directions:

(1) Studying mod A in-depth, such as Auslander-Reiten theory, homological dimensions, triangulated categories, etc, for rep-finite and tame algebras;

(2) Studying nice subcategories of mod A, such as Serre subcategories, wide subcategories, etc, for wild algebras.

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Aim of this talk

To capture **some finite property** in wild cases.

Brick finiteness of algebras

A module M is called a **brick** if $\mathrm{End}_A(M) \simeq K$.

Then, A is said to be

(1) (Chindris-Kinser-Weyman, 2012) **Schur-representation-finite** if there are finitely many bricks of a fixed dimension.

(2) (Demonet-Iyama-Jasso, 2016) **brick-finite** if there are finitely many bricks in the module category of A.

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- (2) $\Rightarrow$ (1) is obvious.
- (1) $\Rightarrow$ (2) is not verified; no counterexample.

Wild, but brick-finite

Set $\Lambda_n = KQ/I_n$ with

$$Q: \ 1 \xrightarrow{\alpha} 2 \circlearrowleft \beta \quad \text{and } I_n : \langle \beta^n, \alpha\beta^2 \rangle, \ n \geqslant 2,$$

the representation type of Λ_n is

- rep-finite if $n \leqslant 5$;
- tame if $n = 6$;
- wild if $n \geqslant 7$.

But, Λ_n admits only 4 bricks for any $n \geqslant 2$.

Known Result

The brick finiteness is known, for example, for

- preprojective algebras of Dynkin type (Mizuno, 2014);
- algebras with radical square zero (Adachi, 2016);
- cycle-finite algebras (Malicki-Skowroński, 2016);
- Brauer graph algebras (Adachi-Aihara-Chan, 2018);
- gentle algebras (Plamondon, 2018);
- (special) biserial algebras (Mousavand, 2019; Schroll-Treffinger-Valdivieso, 2021);
- cluster-tilted algebras (Zito, 2019);
- minimal wild two-point algebras (W., 2019);
- quasi-tilted algebras, locally hereditary algebras, etc., (Aihara-Honma-Miyamoto-W., 2020).

τ-tilting theory

τ-tilting theory was introduced by Adachi, Iyama and Reiten in 2014, as a completion to the classical tilting theory.

So far, τ-tilting theory is related to several different aspects in Representation Theory of Algebras:

- Categorical objects, such as torsion class, silting complex;
- Combinatorial objects, such as brick, semibrick;
- Lattice theory, such as the lattice of torsion classes;
- Geometric objects, such as the modern Brauer-Thrall conjecture, wall-and-chamber structure.

Auslander-Reiten translation

Nakayama functor $\nu(-) : \text{proj } A \to \text{inj } A$

Let M be an A-module with a minimal projective presentation

$$P_1 \xrightarrow{f_1} P_0 \xrightarrow{f_0} M \longrightarrow 0,$$

the **Auslander-Reiten translation** τM is defined by the following exact sequence

$$0 \longrightarrow \tau M \longrightarrow \nu P_1 \xrightarrow{\nu f_1} \nu P_0,$$

that is, $\tau M = \ker \nu f_1$.

Definition 2.1 (Adachi-Iyama-Reiten, 2014)

Let M be a right A-module. Then,

(1) M is called τ-rigid if $\operatorname{Hom}_A(M, \tau M) = 0$.

(2) M is called τ-tilting if M is τ-rigid and $|M| = |A|$.

(3) M is called support τ-tilting if M is a τ-tilting (A/AeA)-module for an idempotent e of A.

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We have

$$\mathrm{i}\tau\text{-rigid}\, A \quad \text{gives} \quad \tau\text{-tilt}\, A \subseteq \mathrm{s}\tau\text{-tilt}\, A \subseteq \tau\text{-rigid}\, A$$

Mutation

Reminder: $M_1 \oplus \cdots \oplus M_j \oplus \cdots \oplus M_n \Rightarrow M_1 \oplus \cdots \oplus M_j^* \oplus \cdots \oplus M_n$.

- add(M): the full subcategory whose objects are direct summands of finite direct sums of copies of M;
- Fac(M): the full subcategory whose objects are factor modules of finite direct sums of copies of M.

Definition 2.2 (AIR, 2014)

Let $M = M_1 \oplus \cdots \oplus M_j \oplus \cdots \oplus M_n$ with $M_j \notin \mathrm{Fac}(M/M_j)$. Take a minimal left add(M/M_j)-approximation π with an exact sequence

$$M_j \xrightarrow{\pi} Z \longrightarrow \mathrm{coker}\ \pi \longrightarrow 0.$$

We call $\mu_j^-(M) := \mathrm{coker}\ \pi \oplus (M/M_j)$ the left mutation of M with respect to M_j, which is again a support τ-tilting A-module.

Mutation Graph

We draw an arrow $M \to \mu_j^-(M)$, it gives a graph $\mathcal{H}(\mathrm{s}\tau\text{-tilt}\, A)$.

For example, $\mathcal{H}(\mathrm{s}\tau\text{-tilt}\, \Lambda_2)$ is displayed as

$$
\begin{array}{ccccccc}
\begin{smallmatrix}1\\2\\2\end{smallmatrix} \oplus \begin{smallmatrix}2\\2\end{smallmatrix} & \longrightarrow & & & & & \begin{smallmatrix}2\\2\end{smallmatrix} \\
\downarrow & & & & & & \downarrow \\
\begin{smallmatrix}1\\2\\2\end{smallmatrix} \oplus \begin{smallmatrix} & & 1\\ 1 & & 2\\ & 2 & \end{smallmatrix} & \longrightarrow & 1 \oplus \begin{smallmatrix} & & 1\\ 1 & & 2\\ & 2 & \end{smallmatrix} & \longrightarrow & 1 & \longrightarrow & 0
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\end{array}.$$

Proposition 2.3 (AIR, 2014)

If the mutation graph $\mathcal{H}(\text{s}\tau\text{-tilt}\, A)$ contains a finite connected component Δ, then $\mathcal{H}(\text{s}\tau\text{-tilt}\, A) = \Delta$.

Connection with brick finiteness

- brick A: the set of bricks in mod A
- fbrick A: the set of bricks M such that the smallest torsion class $\mathsf{T}(M)$ containing M is functorially finite.

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Theorem 2.4 (Demonet-Iyama-Jasso, 2016)

There exists a bijection between $\mathrm{i}\tau\text{-rigid}\,A$ and fbrick A given by

$$X \mapsto X/\mathrm{rad}_B(X),$$

where $B := \mathrm{End}_A(X)$. If $\mathrm{i}\tau\text{-rigid}\,A$ is finite, brick $A =$ fbrick A.

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e.g.,

iτ-rigid Λ_2	$\begin{smallmatrix}1\\2\\2\end{smallmatrix}$	$\begin{smallmatrix}2\\2\end{smallmatrix}$	$\begin{smallmatrix} & & 1\\ 1 & & 2\\ & 2 & \end{smallmatrix}$	1
brick Λ_2	$\begin{smallmatrix}1\\2\\2\end{smallmatrix}$	2	$\begin{smallmatrix}1\\2\end{smallmatrix}$	1

Reduction Theorem

Proposition 2.5 (Demonet-Iyama-Jasso, 2016)

If A is brick-finite, then

(1) A/I is brick-finite, for any two-sided ideal I of A.

(2) eAe is brick-finite, for any idempotent e of A.

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Proposition 2.6 (Eisele-Janssens-Raedschelders, 2018)

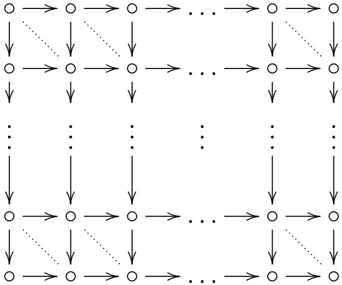
Let I be a two-sided ideal generated by central elements which are contained in the radical of A. Then,

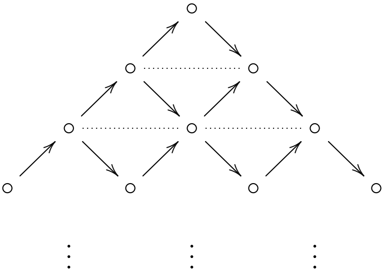
$$\mathsf{s}\tau\text{-tilt}\, A \simeq \mathsf{s}\tau\text{-tilt}\,(A/I).$$

Upper and lower boundary

Upper boundary

Let A be an algebra without loops and oriented cycles. We want to see what happens if A has lots of vertices. For example,





This motivates us to consider simply connected algebras.

Simply connected algebra

Let $A = KQ/I$ without loops and oriented cycles. We consider the fundamental group $\Pi_1(Q, I)$ of A. Then, A is said to be a **simply connected algebra** if, for every bound quiver presentation KQ/I of A, $\Pi_1(Q, I)$ is trivial. (Assem-Skowroński, 1988)

We have the following examples.

(1) All tree algebras are simply connected.

(2) A path algebra KQ is simply connected if and only if Q is a tree. For example, KQ is not simply connected if

$$Q = \begin{array}{ccc} \circ & \longrightarrow & \circ \\ \downarrow & & \downarrow \\ \circ & \longrightarrow & \circ \end{array}.$$

Theorem 3.1 (W., 2019)

Let A be a simply connected algebra. Then,

$$A \text{ is brick-finite} \Leftrightarrow A \text{ is rep-finite.}$$

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Sketch of the proof:

- A: rep-finite $\Rightarrow$ brick-finite, obvious;
- A: rep-infinite

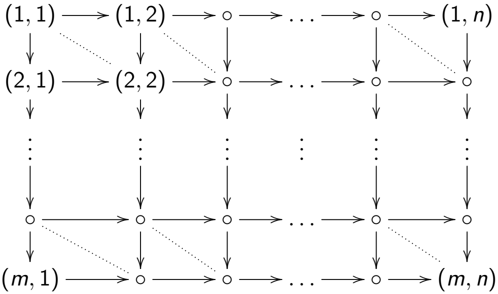
  $\Rightarrow$ there exists an idempotent e of A such that eAe is one of concealed algebras of type $\widetilde{\mathbb{D}}_n$, $\widetilde{\mathbb{E}}_6$, $\widetilde{\mathbb{E}}_7$, $\widetilde{\mathbb{E}}_8$ (Bongartz, 1984);

  $\Rightarrow$ the above eAe is brick-infinite;

  $\Rightarrow$ A is brick-infinite (Proposition 2.4).

Rectangle Quiver

Let $B_{m,n}$ $(m \leqslant n)$ be the algebra given by the following quiver with all possible commutativity relations:



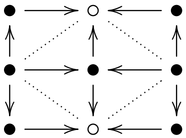
Then, $B_{m,n}$ is brick-finite if and only if

$$(m,n) \in \{(1,n),(2,2),(2,3),(2,4)\}.$$

Tensor product algebras

$$A: \circ \longrightarrow \circ \longleftarrow \circ\,, \qquad B: \circ \longleftarrow \circ \longrightarrow \circ$$

$\Rightarrow A \otimes B:$



<table>
<tr><td colspan="3" rowspan="3">$A \otimes B$: Simply connected</td><td colspan="3">B: Nakayama</td><td colspan="3" rowspan="2">B: non-Nakayama</td></tr>
<tr><td colspan="2">$\mathrm{rad}^2 = 0$</td><td rowspan="2">$\mathrm{rad}^2 \neq 0$</td></tr>
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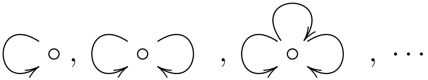
</table>

Lower boundary

A local algebra is always brick-finite, whose quiver is given as



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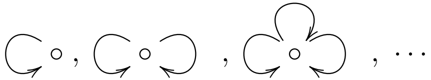
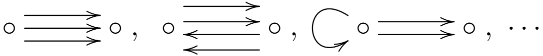
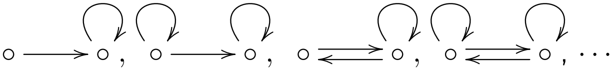


This forces us to focus on $A = KQ/I$ with only two vertices:



or



Two-point algebra

Proposition 3.2

The Kronecker algebra $K(1 \rightrightarrows 2)$ is brick-infinite.

Proof: It is well-known that $K \underset{1}{\overset{\lambda}{\rightrightarrows}} K$ is a brick, for any $\lambda \in K$.

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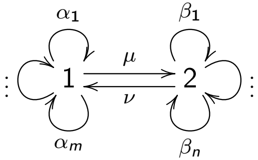
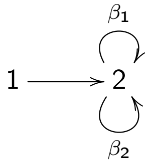
We only need to consider

$Q(m, n) :=$



Theorem 3.3 (W., 2022)

Let $A = KQ(m, n)/I$ be a monomail algebra with $\operatorname{rad}^3 A = 0$. Then, A is brick-finite if and only if it does not have $\Delta = KQ/I$:

Q :


and $I : \langle \beta_1^2, \beta_2^2, \beta_1\beta_2, \beta_2\beta_1 \rangle$,

or its opposite algebra as a quotient algebra.

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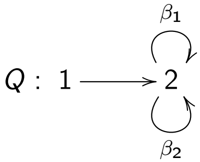

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<u>Sketch of the proof:</u>

(1) sτ-tilt $A \simeq$ sτ-tilt (A/J), $J \subseteq \text{rad } A \cap Z(A)$;

(2) Δ is brick-infinite, using silting theory.

Silting Theory

Proposition 3.4 (AIR, 2014)

There exists a poset isomorphism between sτ-tilt A and 2-silt A, the bijection $\mathcal{F}$ is given by

$$M \longmapsto (P_1 \oplus P \xrightarrow{\binom{f}{0}} P_0)\ ,$$

where (M, P) is the support τ-tilting pair corresponding to M and $P_1 \xrightarrow{f} P_0 \to M \to 0$ is the minimal projective presentation of M.

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A mutation chain: $M^{(1)} \to M^{(2)} \to \cdots \to M^{(k)} \to \cdots$

$$\begin{array}{ccccccccc} \mathcal{F}(M^{(1)}) & \longrightarrow & \mathcal{F}(M^{(2)}) & \longrightarrow \cdots \longrightarrow & \mathcal{F}(M^{(2k-1)}) & \longrightarrow & \mathcal{F}(M^{(2k)}) & \longrightarrow & \cdots \\ \downarrow{\scriptstyle \mathrm{End}} & & \downarrow{\scriptstyle \mathrm{End}} & & \downarrow{\scriptstyle \mathrm{End}} & & \downarrow{\scriptstyle \mathrm{End}} & & \\ B & & B^{\mathrm{op}} & \cdots & B & & B^{\mathrm{op}} & & \cdots \end{array}$$

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- $\circ \xrightarrow{\mu} \circ \circlearrowright \beta$ with $\beta^4 = 0$,
- $\circ \underset{\nu}{\overset{\mu}{\rightleftarrows}} \circ \circlearrowright \beta$ with $\beta^3 = \beta\nu = \nu\mu\nu = \nu\mu\beta^2 = 0$,
- $\alpha \circlearrowright \circ \xrightarrow{\mu} \circ \circlearrowright \beta$ with $\alpha^2 = \beta^2 = 0$,

and their opposite algebras as a quotient algebra.

Application

Derived Equivalence Class

- A is derived equivalent to $B \Leftrightarrow \mathrm{D^b}(\mathrm{mod}\, A) \simeq \mathrm{D^b}(\mathrm{mod}\, B)$

Theorem 4.1 (Ariki-Song-W., 2024)

Let $A_1, A_2, \ldots, A_s$ be pairwise derived equivalent symmetric algebras. Suppose the following conditions hold.

(1) A_i is brick-finite, for all $1 \leq i \leq s$.

(2) $\mathrm{End}(\mathcal{F}(\mu_k^-(A_i))) \in \{A_1, A_2, \ldots, A_s\}$, for any k and all $1 \leq i \leq s$.

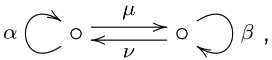
Then, any algebra B which has derived equivalence

$$\mathrm{D^b}(\mathrm{mod}\, B) \cong \mathrm{D^b}(\mathrm{mod}\, A_1)$$

is included in $\{A_1, A_2, \ldots, A_s\}$.

We consider the following quiver:

Q :

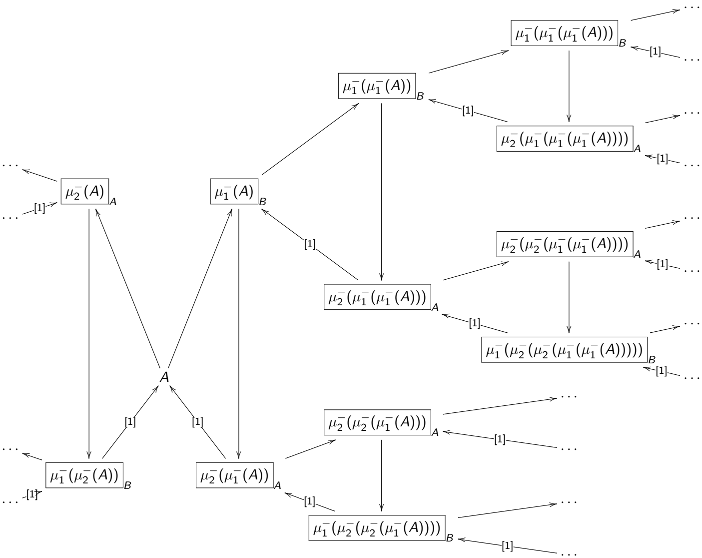


and define

- $A := KQ/\langle\alpha^2, \beta^2 - \nu\mu, \alpha\mu - \mu\beta, \beta\nu - \nu\alpha\rangle$.
- $B := KQ/\langle\alpha^2 - \mu\nu, \beta^2 - \nu\mu, \alpha\mu - \mu\beta, \beta\nu - \nu\alpha, \mu\nu\mu, \nu\mu\nu\rangle$.

Proposition 4.2

If C is derived equivalent to A, then C is isomorphic to A or B.


A
$\mu_1^-(A)_B$
$\mu_2^-(A)_A$
$\mu_1^-(\mu_2^-(A))_B$
$\mu_2^-(\mu_1^-(A))_A$
$\mu_1^-(\mu_1^-(A))_B$
$\mu_2^-(\mu_1^-(\mu_1^-(A)))_A$
$\mu_2^-(\mu_2^-(\mu_1^-(A)))_A$
$\mu_1^-(\mu_2^-(\mu_2^-(\mu_1^-(A))))_B$
$\mu_1^-(\mu_1^-(\mu_1^-(A)))_B$
$\mu_2^-(\mu_1^-(\mu_1^-(\mu_1^-(A))))_A$
$\mu_2^-(\mu_2^-(\mu_1^-(\mu_1^-(A))))_A$
$\mu_1^-(\mu_2^-(\mu_2^-(\mu_1^-(\mu_1^-(A)))))_B$
[1]

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Thank you! Any questions?

- Quiver representation theory;
- Representation type: rep-finite, tame, wild;
- Brick finiteness of algebras;
- τ-tilting theory;

- Simply connected algebras;
- Two-point algebras;
- Silting theory;
- Derived equivalence class.