Representation type of cyclotomic quiver Hecke algebras in affine type A[1]

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[1]This is joint work with Susumu Ariki and Linliang Song.

Outline

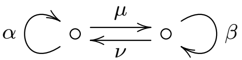
Introduction

Goal of Algebraic Representation Theory

Classify all indecomposable modules of a given algebra A and all morphisms between them, up to isomorphism.

Any (basic, connected) algebra A over an algebraically closed field K is isomorphic to a **bound quiver algebra** KQ/I.

e.g.,



- paths: $(\alpha\mu\beta\nu)^m, (\mu\nu)^n\alpha^k, (\alpha\mu\nu)^k(\mu\beta\nu)^m$, ...

Representaion type of algebra

Theorem (Drozd 1977)

The representation type of any algebra (over K) is exactly one of rep-finite, tame and wild.

An algebra A is said to be

- **rep-finite** if the number of indecomposable modules is finite.
- **tame** if it is not rep-finite, but all indecomposable modules can be organized in a one-parameter family in each dimension.

Otherwise, A is called **wild**.

"The representation type of symmetric algebras is preserved under derived equivalence."

Main result

Main Theorem (Ariki-Song-W. 2023)

Suppose $|\Lambda| \geq 3$. The cyclotomic quiver Hecke algebra $R^\Lambda(\beta)$ of type $A_\ell^{(1)}$ is rep-finite if $\beta \in \mathcal{F}(\Lambda)$, tame if one of the following holds:

- $\beta = \delta$, $\Lambda = k\Lambda_i$, $\ell = 1$ with $t \neq \pm 2$,
- $\beta = \delta$, $\Lambda = k\Lambda_i$, $\ell \geq 2$ with $t \neq (-1)^{\ell+1}$,
- $\beta \in \mathcal{T}(\Lambda)$.

Otherwise, it is wild.

More on Hecke algebras

In the last fifty years, the representation theory of symmetric groups had a close connection with Lie theory via **categorification**.

- Hecke algebras of Coxeter groups, i.e., of type A, B, D, etc.
- Cyclotomic Hecke algebras (a.k.a. Ariki-Koike algebras). See [Ariki-Koike, 1994], [Broue-Malle, 1993], [Cherednik 1987].
- Cyclotomic quiver Hecke algebras (a.k.a. Cyclotomic KLR algebras). See [Khovanov-Lauda, 2009], [Rouquier, 2008].

Many classes of algebras arise in this process, whose representation type is completely determined, in particular, for

(1) Hecke alg's in type A, B (Erdmann-Nakano 2001, Ariki-Mathas 2004);

(2) Cyclotomic quiver Hecke alg's of level 1 in affine type A, C, D (Ariki-Iijima-Park 2014, 2015); of level 2 in affine type A (Ariki 2017);

(3) Schur/q-Schur/Borel-Schur/infinitesimal-Schur alg's (Xi 1993, Erdmann 1993, Doty-Erdmann-Martin 1999, Erdmann-Nakano 2001, etc);

(4) block alg's of category $\mathcal{O}$; (Futorny-Nakano-Pollack 1999, Boe-Nakano 2005, etc)

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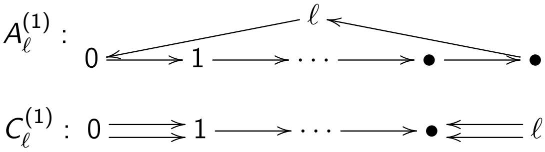
Cyclotomic quiver Hecke algebras

Lie theoretic data

Let $I = \{0, 1, ..., \ell\}$ be an index set. Recall that



$$+B_\ell^{(1)}, D_\ell^{(1)}, A_{2\ell}^{(2)}, A_{2\ell-1}^{(2)}, D_{\ell+1}^{(2)}, E_6^{(1)}, E_7^{(1)}, E_8^{(1)}, F_4^{(1)}, G_2^{(1)}, E_6^{(2)}, D_4^{(3)}.$$

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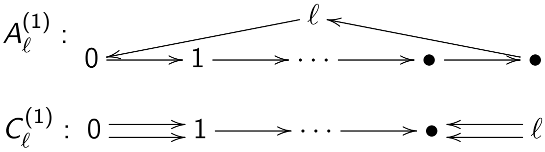

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Set $n_{ij} := \#(i \to j)$.

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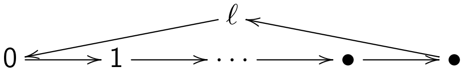
$A_\ell^{(1)}$:



$C_\ell^{(1)}$:

$$+B_\ell^{(1)}, D_\ell^{(1)}, A_{2\ell}^{(2)}, A_{2\ell-1}^{(2)}, D_{\ell+1}^{(2)}, E_6^{(1)}, E_7^{(1)}, E_8^{(1)}, F_4^{(1)}, G_2^{(1)}, E_6^{(2)}, D_4^{(3)}.$$

Set $n_{ij} := \#(i \to j)$. We define the **Cartan matrix** $\mathrm{A} = (a_{ij})_{i,j \in I}$ by

$$a_{ii} = 2, \quad a_{ij} = \begin{cases} -n_{ij} & \text{if } n_{ij} > n_{ji} \\ -1 & \text{if } n_{ij} < n_{ji} \quad (i \neq j). \\ -n_{ij} - n_{ji} & \text{otherwise} \end{cases}$$

Let $(\mathsf{A}, P, \Pi, P^\vee, \Pi^\vee)$ be the **Cartan datum** in type $X^{(1)}$, where

- $P = \mathbb{Z}\Lambda_0 \oplus \mathbb{Z}\Lambda_1 \oplus \cdots \oplus \mathbb{Z}\Lambda_\ell \oplus \mathbb{Z}\delta$ is the weight lattice;
- $\Pi = \{\alpha_i \mid 0 \le i \le \ell\} \subset P$ is the set of simple roots;
- $P^\vee = \mathrm{Hom}(P, \mathbb{Z})$ is the coweight lattice;
- $\Pi^\vee = \{h_i \mid 0 \le i \le \ell\} \subset P^\vee$ is the set of simple coroots.

We have

$$\langle h_i, \alpha_j \rangle = a_{ij}, \quad \langle h_i, \Lambda_j \rangle = \delta_{ij} \quad \text{for all } 0 \le i, j \le \ell.$$

The null root is δ, e.g.,

$$\delta = \begin{cases} \alpha_0 + \alpha_1 + \cdots + \alpha_\ell & \text{if } X = A_\ell, \\ \alpha_0 + 2(\alpha_1 + \cdots + \alpha_{\ell-1}) + \alpha_\ell & \text{if } X = C_\ell. \end{cases}$$

Quiver Hecke algebra

The **quiver Hecke algebra** $R(n)$ associated with $(Q_{i,j}(u,v))_{i,j\in I}$ is the K-algebra generated by

$$\{e(\nu) \mid \nu = (\nu_1, \nu_2, \ldots, \nu_n) \in I^n\}, \quad \{x_i \mid 1 \le i \le n\}, \quad \{\psi_j \mid 1 \le j \le n-1\},$$

subject to the following relations:

(1) $e(\nu)e(\nu') = \delta_{\nu,\nu'} e(\nu),\ \sum_{\nu \in I^n} e(\nu) = 1,\ x_i x_j = x_j x_i,\ x_i e(\nu) = e(\nu) x_i.$

(2) $\psi_i e(\nu) = e(s_i(\nu))\psi_i,\ \psi_i\psi_j = \psi_j\psi_i$ if $|i-j| > 1$.

(3) $\psi_i^2 e(\nu) = Q_{\nu_i,\nu_{i+1}}(x_i, x_{i+1}) e(\nu)$.

(4) $(\psi_i x_j - x_{s_i(j)}\psi_i)e(\nu) = \begin{cases} -e(\nu) & \text{if } j = i \text{ and } \nu_i = \nu_{i+1}, \\ e(\nu) & \text{if } j = i+1 \text{ and } \nu_i = \nu_{i+1}, \\ 0 & \text{otherwise.} \end{cases}$

(5) $(\psi_{i+1}\psi_i\psi_{i+1} - \psi_i\psi_{i+1}\psi_i)e(\nu) = \begin{cases} \frac{Q_{\nu_i,\nu_{i+1}}(x_i,x_{i+1}) - Q_{\nu_i,\nu_{i+1}}(x_{i+2},x_{i+1})}{x_i - x_{i+2}} e(\nu) & \text{if } \nu_i = \nu_{i+2}, \\ 0 & \text{otherwise.} \end{cases}$

A family of polynomials in affine type A

Fix $t \in K$ if $\ell = 1$ and $0 \neq t \in K$ if $\ell \geq 2$.

For $i, j \in I$, we take $Q_{i,j}(u, v) \in K[u, v]$ such that $Q_{i,i}(u, v) = 0$, $Q_{i,j}(u, v) = Q_{j,i}(v, u)$ and if $\ell \geq 2$,

$$\begin{aligned} Q_{i,i+1}(u, v) &= u + v \text{ if } 0 \leq i < \ell, \\ Q_{\ell,0}(u, v) &= u + tv, \\ Q_{i,j}(u, v) &= 1 \text{ if } j \not\equiv_{\ell+1} i, i \pm 1. \end{aligned}$$

If $\ell = 1$, we take $Q_{0,1}(u, v) = u^2 + tuv + v^2$.

Cyclotomic quiver Hecke algebras

Set

$$\Lambda = a_0\Lambda_0 + a_1\Lambda_1 + \cdots + a_\ell\Lambda_\ell \in P^+, \; a_i \in \mathbb{Z}_{\geq 0}.$$

The **cyclotomic quiver Hecke algebra** $R^\Lambda(n)$ is defined as the quotient of $R(n)$ modulo the relation

$$x_1^{\langle h_{\nu_1}, \Lambda \rangle} e(\nu) = 0.$$

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Set

$$\beta = b_0\alpha_0 + b_1\alpha_1 + \cdots + b_\ell\alpha_\ell \in Q^+, \ b_i \in \mathbb{Z}_{\geq 0},$$

with $|\beta| = b_1 + \cdots + b_\ell = n$, we define

$$R^\Lambda(\beta) := e(\beta)R^\Lambda(n)e(\beta),$$

where $e(\beta) := \sum_{\nu \in I^\beta} e(\nu)$ with $I^\beta = \left\{\nu = (\nu_1, \nu_2, \ldots, \nu_n) \in I^n \mid \sum_{i=1}^{n} \alpha_{\nu_i} = \beta\right\}$.

Representation type of $R^{\Lambda}(\beta)$

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$$\{\mu - m\delta \mid \mu \in \max^+(\Lambda), m \in \mathbb{Z}_{\geq 0}\},$$

which is the W-orbit of the set $P(\Lambda)$ of weights of $V(\Lambda)$, where W is the affine symmetric group and $V(\Lambda)$ is the integrable highest weight module of the quantum group. See: [Chuang-Rouquier, 2008].

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- A weight $\mu \in P(\Lambda)$ is maximal if $\mu + \delta \notin P(\Lambda)$. We define

$$\max^+(\Lambda) := \{\mu \in P^+ \mid \mu \text{ is maximal}\}.$$

$\max^+(\Lambda)$

Theorem (Kim-Oh-Oh 2020)

There is a bijection $\phi_\Lambda : \max^+(\Lambda) \to P_k^+(\Lambda)$.

Set $\Lambda = a_{i_1}\Lambda_{i_1} + a_{i_2}\Lambda_{i_2} + \cdots + a_{i_n}\Lambda_{i_n} \in P^+$. Then,

$$|\Lambda| := a_{i_1} + \cdots + a_{i_j} \quad \text{and} \quad \mathrm{ev}(\Lambda) := i_1 + \cdots + i_n.$$

In type $A_\ell^{(1)}$, we have

$$P_k^+(\Lambda) := \left\{ \Lambda' \in P^+ \mid |\Lambda| = |\Lambda'| = k, \mathrm{ev}(\Lambda) \equiv_{\ell+1} \mathrm{ev}(\Lambda') \right\}.$$

In type $C_\ell^{(1)}$, we have

$$P_k^+(\Lambda) := \left\{ \Lambda' \in P^+ \mid |\Lambda| = |\Lambda'| = k, \mathrm{ev}(\Lambda) \equiv_2 \mathrm{ev}(\Lambda') \right\}.$$

Recall that $\langle h_i, \Lambda_j \rangle = \delta_{ij}$. We define $y_i := \langle h_i, \Lambda - \Lambda' \rangle$ and

$$Y_{\Lambda'} := (y_0, y_1, \dots, y_\ell) \in \mathbb{Z}^{\ell+1}.$$

Theorem (Ariki-Song-W. 2023)

The bijection $\phi_\Lambda^{-1} : P_k^+(\Lambda) \to \max^+(\Lambda)$ is given by

$$\Lambda' \mapsto \Lambda - \sum_{i=0}^{\ell} x_i \alpha_i,$$

where $X = (x_0, x_1, \dots, x_\ell)$ is the unique solution of $AX^t = Y_{\Lambda'}^t$ satisfying

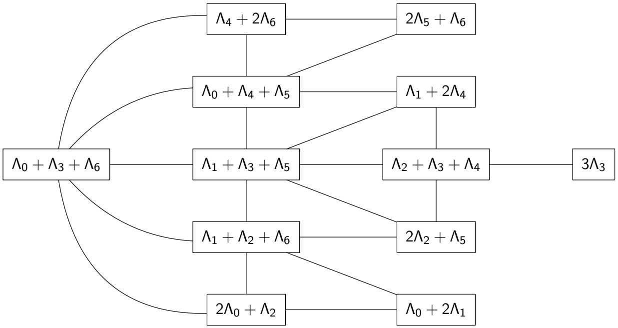
$$x_i \geq 0 \quad \text{and} \quad \min\{x_i\} = 0.$$

Structure of $P_k^+(\Lambda)$

A finite connected quiver

e.g., $P_3^+(\Lambda_0 + \Lambda_3 + \Lambda_6)$ in type $A_6^{(1)}$ consists of $\Lambda_0 + \Lambda_3 + \Lambda_6$, $\Lambda_1 + \Lambda_2 + \Lambda_6$, $\Lambda_1 + \Lambda_3 + \Lambda_5$, $\Lambda_0 + \Lambda_4 + \Lambda_5$, $\Lambda_2 + \Lambda_3 + \Lambda_4$, $2\Lambda_0 + \Lambda_2$, $\Lambda_4 + 2\Lambda_6$, $2\Lambda_5 + \Lambda_6$, $\Lambda_0 + 2\Lambda_1$, $2\Lambda_2 + \Lambda_5$, $\Lambda_1 + 2\Lambda_4$, $2\Lambda_0 + \Lambda_2$, $3\Lambda_3$.

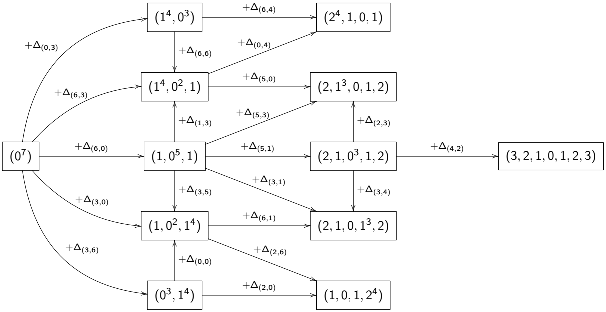
If $j \not\equiv_{\ell+1} i - 1$, we draw $\boxed{\Lambda_i + \Lambda_j + \tilde{\Lambda}}$ — $\boxed{\Lambda_{i-1} + \Lambda_{j+1} + \tilde{\Lambda}}$.
e.g.,



We define

$$\Delta_{i,j} = \begin{cases} (0^i, 1^{j-i+1}, 0^{\ell-j}) & \text{if } i \leq j, \\ (1^{j+1}, 0^{i-j-1}, 1^{\ell-i+1}) & \text{if } i > j. \end{cases}$$

The unique solution of $AX^t = Y^t_{\Lambda'}$ is given by $\min(X_{\Lambda'} + \Delta_{i,j}) = 0$.
e.g.,



Rule to draw arrows

Let Δ^+_{fin} be the set of positive roots of the root system of type X.

Then, the set $\Delta^+_{\mathrm{fin}} \sqcup (\delta - \Delta^+_{\mathrm{fin}})$ gives all arrows $\Lambda' \longrightarrow \Lambda''$.

- If $X = A_\ell$, $\Delta^+_{\mathrm{fin}} = \{\epsilon_i - \epsilon_j \mid 1 \le i < j \le \ell + 1\}$.
- If $X = B_\ell$, $\Delta^+_{\mathrm{fin}} = \{\epsilon_i \mid 1 \le i \le \ell\} \sqcup \{\epsilon_i \pm \epsilon_j \mid 1 \le i < j \le \ell\}$.
- If $X = C_\ell$, $\Delta^+_{\mathrm{fin}} = \{2\epsilon_i \mid 1 \le i \le \ell\} \sqcup \{\epsilon_i \pm \epsilon_j \mid 1 \le i < j \le \ell\}$.
- If $X = D_\ell$, $\Delta^+_{\mathrm{fin}} = \{\epsilon_i \pm \epsilon_j \mid 1 \le i < j \le \ell\}$.

Key Lemmas

Lemma 1

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Lemma 2

Suppose $\Lambda = \bar{\Lambda} + \tilde{\Lambda}$. There is a directed path

$$\Lambda^{(1)} \longrightarrow \Lambda^{(2)} \longrightarrow \cdots \longrightarrow \Lambda^{(m)} \in \vec{C}(\bar{\Lambda})$$

if and only if there is a directed path

$$\Lambda^{(1)} + \tilde{\Lambda} \longrightarrow \Lambda^{(2)} + \tilde{\Lambda} \longrightarrow \cdots \longrightarrow \Lambda^{(m)} + \tilde{\Lambda} \in \vec{C}(\Lambda).$$

Lemma 3

Suppose that there is an arrow $\Lambda' \longrightarrow \Lambda''$ in $\vec{C}(\Lambda)$. If $R^{\Lambda}(\beta_{\Lambda'})$ is representation-infinite (resp. wild), then so is $R^{\Lambda}(\beta_{\Lambda''})$.

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Lemma 4

Write $\Lambda = \bar{\Lambda} + \tilde{\Lambda}$. If $R^{\bar{\Lambda}}(\beta)$ is representation-infinite (resp. wild), then $R^{\Lambda}(\beta)$ is representation-infinite (resp. wild).

Rep-finite and tame sets in affine type A

Set $i_0 := i_h$, $i_{h+1} := i_1$ and write

$$\Lambda = m_{i_1}\Lambda_{i_1} + \cdots + m_{i_j}\Lambda_{i_j} + m_{i_{j+1}}\Lambda_{i_{j+1}} + \cdots + m_{i_h}\Lambda_{i_h}$$

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For any $1 \leq j \leq h$, we define

$$F(\Lambda)_0 := \{\Lambda_{i_j,i_j} \mid m_{i_j} = 2\}$$
$$F(\Lambda)_1 := \{\Lambda_{i_j,i_{j+1}} \mid m_{i_j} = 1, m_{i_{j+1}} = 1\}$$
$$T(\Lambda)_1 := \{\Lambda_{i_j,i_{j+1}} \mid m_{i_j} = 1, m_{i_{j+1}} > 1 \text{ or } m_{i_j} > 1, m_{i_{j+1}} = 1\}$$
$$T(\Lambda)_2 := \{(\Lambda_{i_j,i_j})_{i_j-1,i_j+1} \mid m_{i_j} = 2, i_{j-1} \not\equiv_e i_j - 1, i_{j+1} \not\equiv_e i_j + 1\} \text{ if char } K \neq 2$$
$$T(\Lambda)_3 := \{(\Lambda_{i_j,i_j})_{i_j,i_j+1 \text{ or } i_j-1,i_j} \mid m_{i_j} = 3, i_{j+1} \not\equiv_e i_j + 1 \text{ or } i_{j-1} \not\equiv_e i_j - 1\} \text{ if char } K \neq 3$$
$$T(\Lambda)_4 := \{(\Lambda_{i_j,i_j})_{i_j,i_j} \mid m_{i_j} = 4\} \text{ if char } K \neq 2$$
$$T(\Lambda)_5 := \{(\Lambda_{i_j,i_j})_{i_p,i_p} \mid m_{i_j} = m_{i_p} = 2, i_p \not\equiv_e i_j \pm 1, j \neq p\}$$

Set

$$\mathcal{F}(\Lambda) = \{\beta_{\Lambda'} \mid \Lambda' \in \{\Lambda\} \cup F(\Lambda)_0 \cup F(\Lambda)_1\},$$

$$\mathcal{T}(\Lambda) = \{\beta_{\Lambda'} \mid \Lambda' \in \cup_{1 \leq j \leq 5} T(\Lambda)_j\}.$$

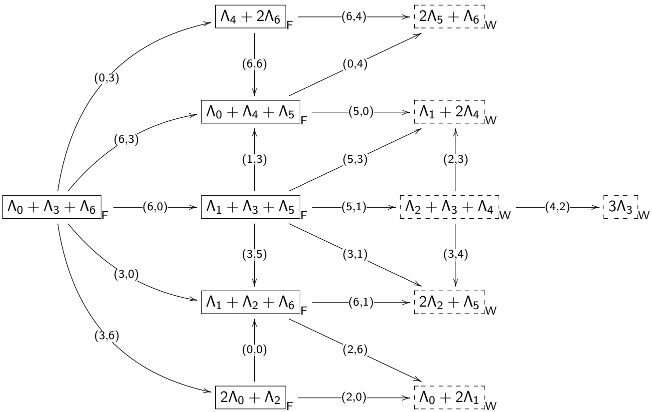
Theorem (Ariki-Song-W. 2023)

Suppose $\mathrm{le}(\Lambda) \geq 3$. Then, $R^{\Lambda}(\beta)$ is representation-finite if $\beta \in \mathcal{F}(\Lambda)$, tame if one of the following holds:

- $\beta = \delta$, $\Lambda = k\Lambda_i$, $\ell = 1$ with $t \neq \pm 2$,
- $\beta = \delta$, $\Lambda = k\Lambda_i$, $\ell \geq 2$ with $t \neq (-1)^{\ell+1}$,
- $\beta \in \mathcal{T}(\Lambda)$.

Otherwise, it is wild.

e.g., rep-type of $\vec{C}(\Lambda_0 + \Lambda_3 + \Lambda_6)$ in type $A_6^{(1)}$ is displayed as



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Thank you! Any questions?

- Bound quiver algebras;
- Representation type: rep-finite, tame, wild.

- Lie theoretic data and Cartan datum;
- Quiver Hecke algebras;
- Cyclotomic quiver Hecke algebras;
- Representation type of $R^{\Lambda}(\beta)$;
- $\max^{+}(\Lambda)$ and $P_k^{+}(\Lambda)$;
- Rep-finite and tame sets in affine type A.