# Representation type of cyclotomic quiver Hecke algebras[1]

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[1]Collaborations with Susumu Ariki, Berta Hudak, and Linliang Song.

# Outline

# Background

# Cyclotomic quiver Hecke algebra

## a.k.a. cyclotomic Khovanov-Lauda-Rouquier algebra

- $U_q(\mathfrak{g})$: the quantum group of certain Kac-Moody algebra $\mathfrak{g}$
- $V(\Lambda)$: the integrable highest weight $U_q(\mathfrak{g})$-module with the highest weight $\Lambda$
- $\mathcal{R}^\Lambda$: the cyclotomic quiver Hecke algebra

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- $\mathcal{R}^\Lambda$: the cyclotomic quiver Hecke algebra

| **Lie Theory** | **Representation Theory** |
|---|---|
| Weight spaces of $V(\Lambda)$ | Blocks of $\mathcal{R}^\Lambda$ |
| Crystal graph of $V(\Lambda)$ | Socle branching rule for $\mathcal{R}^\Lambda$ |
| Canonical basis in $V(\Lambda)$ over $\mathbb{C}$ | Indecom. projective $\mathcal{R}^\Lambda$-modules |
| Action of the Weyl group of $\mathfrak{g}$ on $V(\Lambda)$ | Derived equivalences between blocks of $\mathcal{R}^\Lambda$ |

### Goal of Algebraic Representation Theory

Classify all indecomposable modules of a given algebra $A$ and all morphisms between them, up to isomorphism.

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An algebra $A$ is said to be

- **rep-finite** if the number of indecomposable modules is finite.
- **tame** if $A$ is not rep-finite, but all indecomposable modules can be organized in a one-parameter family in each dimension.
- **wild** if there exists a faithful exact $K$-linear functor from the module category of $K\langle x, y\rangle$ to $\operatorname{mod} A$.

# Representation type of algebra

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It leads to two directions:

**(1)** Studying $\operatorname{mod} A$ in-depth, such as Auslander-Reiten theory, homological dimensions, triangulated categories, etc, for rep-finite and tame algebras;

**(2)** Studying nice subcategories of $\operatorname{mod} A$, such as Serre subcategories, wide subcategories, etc, for wild algebras.

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*"The representation type of symmetric algebras is preserved under derived equivalence."* *(Rickard 1991, Krause 1998)*

# Cyclotomic quiver Hecke algebras

# Lie theoretic data

Let $(\mathsf{A}, P, \Pi, P^\vee, \Pi^\vee)$ be the **Cartan datum** of type $X^{(1)}$, where

- $\mathsf{A} = (a_{ij})_{1 \le i.j \le \ell}$ is the Cartan matrix;
- $P = \mathbb{Z}\Lambda_0 \oplus \mathbb{Z}\Lambda_1 \oplus \cdots \oplus \mathbb{Z}\Lambda_\ell \oplus \mathbb{Z}\delta$ is the weight lattice;
- $\Pi = \{\alpha_i \mid 0 \le i \le \ell\}$ is the set of simple roots;
- $P^\vee = \mathrm{Hom}(P, \mathbb{Z})$ is the coweight lattice;
- $\Pi^\vee = \{h_i \mid 0 \le i \le \ell\}$ is the set of simple coroots.

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We have

$$\langle h_i, \alpha_j \rangle = a_{ij}, \quad \langle h_i, \Lambda_j \rangle = \delta_{ij} \quad \text{for } 0 \le i, j \le \ell.$$

The null root is $\delta$, e.g.,

$$\delta = \begin{cases} \alpha_0 + \alpha_1 + \cdots + \alpha_\ell & \text{if } X = A_\ell, \\ \alpha_0 + 2(\alpha_1 + \cdots + \alpha_{\ell-1}) + \alpha_\ell & \text{if } X = C_\ell. \end{cases}$$

# Cyclotomic quiver Hecke algebra

The **cyclotomic quiver Hecke algebra** $R^{\Lambda}(\beta)$ with

$$\Lambda = a_0\Lambda_0 + \cdots + a_\ell\Lambda_\ell,\ \beta = b_0\alpha_0 + \cdots + b_\ell\alpha_\ell, \quad a_i, b_i \in \mathbb{Z}_{\geq 0},$$

is the $K$-algebra generated by

$$\{e(\nu) \mid \nu = (\nu_1, \nu_2, \ldots, \nu_n) \in I^n\}, \quad \{x_i \mid 1 \leq i \leq n\}, \quad \{\psi_j \mid 1 \leq j \leq n-1\},$$

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subject to the following relations:

- $e(\nu)e(\nu') = \delta_{\nu,\nu'}e(\nu), \quad \sum_{\nu\in I^n} e(\nu) = 1.$
- $x_1^{\langle h_{\nu_1},\Lambda\rangle}e(\nu) = 0, \quad x_ie(\nu) = e(\nu)x_i, \quad x_ix_j = x_jx_i.$
- $\psi_i^2e(\nu) = Q_{\nu_i,\nu_{i+1}}(x_i, x_{i+1})e(\nu), \quad \psi_ie(\nu) = e(s_i(\nu))\psi_i, \quad \psi_i\psi_j = \psi_j\psi_i$ if $|i-j| > 1$.
- $(\psi_ix_j - x_{s_i(j)}\psi_i)e(\nu) = \begin{cases} -e(\nu) & \text{if } j = i \text{ and } \nu_i = \nu_{i+1}, \\ e(\nu) & \text{if } j = i+1 \text{ and } \nu_i = \nu_{i+1}, \\ 0 & \text{otherwise.} \end{cases}$
- $(\psi_{i+1}\psi_i\psi_{i+1} - \psi_i\psi_{i+1}\psi_i)e(\nu) = \begin{cases} \frac{Q_{\nu_i,\nu_{i+1}}(x_i,x_{i+1})-Q_{\nu_i,\nu_{i+1}}(x_{i+2},x_{i+1})}{x_i-x_{i+2}}e(\nu) & \text{if } \nu_i = \nu_{i+2}, \\ 0 & \text{otherwise.} \end{cases}$

**(1)** $R^{\Lambda}(\beta)$ is a finite-dimensional symmetric algebra, see [Shan-Varagnolo-Vasserot, 2017].

**(2)** $R^{\Lambda}(\beta) \sim_{\text{derived}} R^{\Lambda}(\beta')$ if both $\Lambda - \beta$ and $\Lambda - \beta'$ lie in

$$\{\mu - m\delta \mid \mu \in \max^{+}(\Lambda), m \in \mathbb{Z}_{\geq 0}\},$$

see [Chuang-Rouquier, 2008].

**(3)** There is a bijection $\phi_{\Lambda} = \iota_{\Lambda} \circ \bar{\ }: \max^{+}(\Lambda) \to P_k^{+}(\Lambda)$, see [Kim-Oh-Oh, 2020].

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Set $\Lambda = m_{i_1}\Lambda_{i_1} + m_{i_2}\Lambda_{i_2} + \cdots + m_{i_n}\Lambda_{i_n}, m_{i_j} \neq 0$. Then,

$$|\Lambda| := m_{i_1} + \cdots + m_{i_j} \quad \text{and} \quad \text{ev}(\Lambda) := i_1 + \cdots + i_n.$$

In type $A_\ell^{(1)}$,

$$P_k^+(\Lambda) := \left\{\Lambda' \in P^+ \mid |\Lambda| = |\Lambda'| = k, \text{ev}(\Lambda) \equiv_{\ell+1} \text{ev}(\Lambda') \right\}.$$

Recall that $\langle h_i, \Lambda_j \rangle = \delta_{ij}$. We define $y_i := \langle h_i, \Lambda - \Lambda' \rangle$ and

$$Y_{\Lambda'} := (y_0, y_1, \ldots, y_\ell) \in \mathbb{Z}^{\ell+1}.$$

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### Theorem (Ariki-Song-W., 2023)

The equation $AX^t = Y^t_{\Lambda'}$ has a unique solution $X = (x_0, x_1, \ldots, x_\ell)$ satisfying

$$x_i \geq 0 \quad \text{and} \quad \min\{x_i - \delta\} < 0.$$

Set $\beta_{\Lambda'} := x_0\alpha_0 + x_1\alpha_1 + \cdots + x_\ell\alpha_\ell$. Then,

$$\begin{aligned} \phi_\Lambda^{-1} : P_k^+(\Lambda) &\to \max{}^+(\Lambda) \\ \Lambda' &\mapsto \Lambda - \beta_{\Lambda'}. \end{aligned}$$

# Constructions in affine type A

$$\Lambda' = \Lambda_i + \Lambda_j + \tilde{\Lambda} \in P_k^+(\Lambda) \Rightarrow \Lambda'_{i^-,j^+} := \Lambda_{i-1} + \Lambda_{j+1} + \tilde{\Lambda} \in P_k^+(\Lambda)$$

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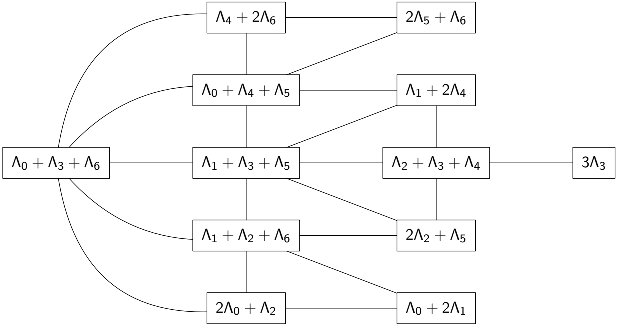
e.g., $P_3^+(\Lambda_0 + \Lambda_3 + \Lambda_6)$ in type $A_6^{(1)}$



We define

$$\Delta_{i^-,j^+} := \begin{cases} (0^i, 1^{j-i+1}, 0^{\ell-j}) & \text{if } i \leq j, \\ (1^{j+1}, 0^{i-j-1}, 1^{\ell-i+1}) & \text{if } i > j. \end{cases}$$

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We draw an arrow $\Lambda' \longrightarrow \Lambda'_{i^-,j^+}$ if

$$X_{\Lambda'} + \Delta_{i^-,j^+} = X_{\Lambda'_{i^-,j^+}}$$

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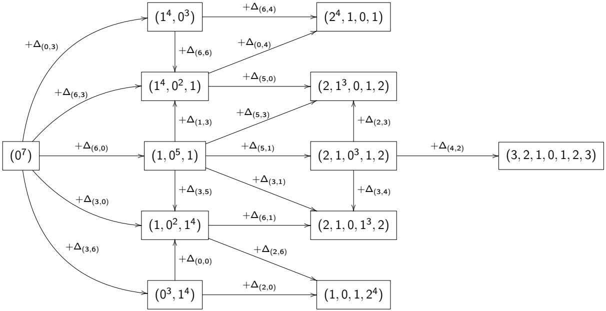

# Key Lemmas

**Lemma 1**

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**Lemma 2**

Suppose $\Lambda = \bar{\Lambda} + \tilde{\Lambda}$. There is a directed path

$$\Lambda^{(1)} \longrightarrow \Lambda^{(2)} \longrightarrow \cdots \longrightarrow \Lambda^{(m)} \in \vec{C}(\bar{\Lambda})$$

if and only if there is a directed path

$$\Lambda^{(1)} + \tilde{\Lambda} \longrightarrow \Lambda^{(2)} + \tilde{\Lambda} \longrightarrow \cdots \longrightarrow \Lambda^{(m)} + \tilde{\Lambda} \in \vec{C}(\Lambda).$$

**Lemma 3**

Write $\Lambda = \bar{\Lambda} + \tilde{\Lambda}$. If $R^{\bar{\Lambda}}(\beta)$ is representation-infinite (resp. wild), then $R^{\Lambda}(\beta)$ is representation-infinite (resp. wild).

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**Lemma 4**

Suppose that there is an arrow $\Lambda' \longrightarrow \Lambda''$ in $\vec{C}(\Lambda)$. If $R^{\Lambda}(\beta_{\Lambda'})$ is representation-infinite (resp. wild), then so is $R^{\Lambda}(\beta_{\Lambda''})$.

# Rep-finite and tame sets in affine type A

Set $i_0 := i_h$, $i_{h+1} := i_1$ and write

$$\Lambda = m_{i_1}\Lambda_{i_1} + \cdots + m_{i_j}\Lambda_{i_j} + m_{i_{j+1}}\Lambda_{i_{j+1}} + \cdots + m_{i_h}\Lambda_{i_h}$$

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For any $1 \le j \le h$, we define

$$F(\Lambda)_0 := \left\{\Lambda_{i_j^-, i_j^+} \mid m_{i_j} = 2\right\}$$
$$F(\Lambda)_1 := \left\{\Lambda_{i_j^-, i_{j+1}^+} \mid m_{i_j} = 1, m_{i_{j+1}} = 1\right\}$$
$$T(\Lambda)_1 := \left\{\Lambda_{i_j^-, i_{j+1}^+} \mid m_{i_j} = 1, m_{i_{j+1}} > 1 \text{ or } m_{i_j} > 1, m_{i_{j+1}} = 1\right\}$$
$$T(\Lambda)_2 := \left\{(\Lambda_{i_j^-, i_j^+})_{(i_j-1)^-, (i_j+1)^+} \mid m_{i_j} = 2, i_{j-1} \not\equiv_{\ell+1} i_j - 1, i_{j+1} \not\equiv_{\ell+1} i_j + 1\right\} \text{ if } \operatorname{char} K \neq 2$$
$$T(\Lambda)_3 := \left\{(\Lambda_{i_j^-, i_j^+})_{i_j^-, (i_j+1)^+ \text{ or } (i_j-1)^-, i_j^+} \mid m_{i_j} = 3, i_{j+1} \not\equiv_{\ell+1} i_j + 1 \text{ or } i_{j-1} \not\equiv_{\ell+1} i_j - 1\right\}$$
$$\text{if } \operatorname{char} K \neq 3$$
$$T(\Lambda)_4 := \left\{(\Lambda_{i_j^-, i_j^+})_{i_j^-, i_j^+} \mid m_{i_j} = 4\right\} \text{ if } \operatorname{char} K \neq 2$$
$$T(\Lambda)_5 := \left\{(\Lambda_{i_j^-, i_j^+})_{i_p^-, i_p^+} \mid m_{i_j} = m_{i_p} = 2, i_p \not\equiv_{\ell+1} i_j \pm 1, j \neq p\right\}$$

Set

$$\mathcal{F}(\Lambda) := \{\beta_{\Lambda'} \mid \Lambda' \in \{\Lambda\} \cup F(\Lambda)_0 \cup F(\Lambda)_1\},$$
$$\mathcal{T}(\Lambda) := \{\beta_{\Lambda'} \mid \Lambda' \in \cup_{1\leq j\leq 5} T(\Lambda)_j\}.$$

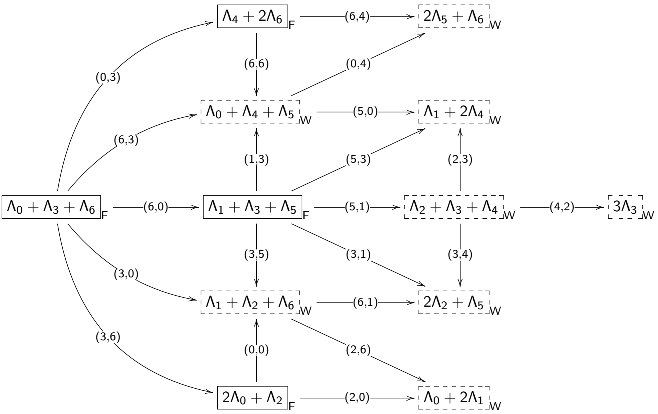
**Theorem (Ariki-Song-W., 2023)**

Suppose $|\Lambda| \geq 3$. Then, $R^{\Lambda}(\beta)$ is representation-finite if $\beta \in \mathcal{F}(\Lambda)$, tame if one of the following holds:

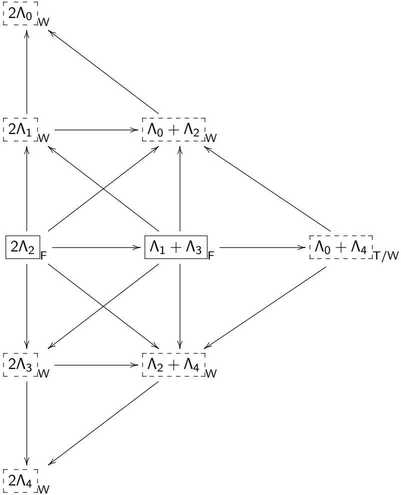
- $\beta = \delta$, $\Lambda = k\Lambda_i$, $\ell = 1$ with $t \neq \pm 2$,
- $\beta = \delta$, $\Lambda = k\Lambda_i$, $\ell \geq 2$ with $t \neq (-1)^{\ell+1}$,
- $\beta \in \mathcal{T}(\Lambda)$.

Otherwise, it is wild.

e.g., rep-type of $\vec{C}(\Lambda_0+\Lambda_3+\Lambda_6)$ in type $A_6^{(1)}$ is displayed as



e.g., rep-type of $\vec{C}(2\Lambda_2)$ in type $C_4^{(1)}$ is displayed as



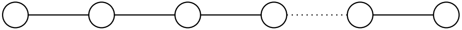
# Derived Equivalence Class

# Affine Type $\mathbb{A}$

Let $R^\Lambda(\beta)$ be the cyclotomic quiver Hecke algebra of type $A_\ell^{(1)}$.

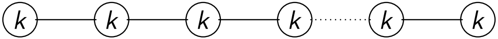
**Theorem (Ariki-Song-W., 2023)**

**(1)** If $R^\Lambda(\beta)$ is representation-finite, then it is derived equivalent to either $K[X]/(X^m)$ for $m \geq 1$ or a Brauer tree algebra whose Brauer tree is displayed as

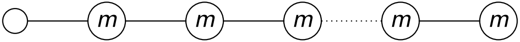


**(2)** If $R^\Lambda(\beta)$ is tame, then it is derived equivalent to one of

- $K[X,Y]/(X^3 - Y^3, XY)$, $K[X,Y]/(X^4 - Y^2, XY)$, $K[X,Y]/(X^2, Y^2)$, $K[X,Y]/(X^k - Y^k, XY)$ for $k \geq 3$.
- Brauer graph algebra associated with




  where $k = |\Lambda|$ and $\#\text{vertices} = \ell + 1$.
- Brauer graph algebra associated with




  where $m$ and $\#$vertices could be calculated explicitly.

# Brauer graph algebra

Let $A$ be a Brauer graph algebra with Brauer graph $\Gamma_A$.

**Theorem (Antipov-Zvonareva, 2022)**

If $B$ is derived equivalent to $A$, then $B$ is Morita equivalent to a Brauer graph algebra.

**Theorem (Opper-Zvonareva, 2022)**

$A \sim_{\text{derived}} B$ if and only if the following conditions hold.

**(1)** $\Gamma_A$ and $\Gamma_B$ share the same number of vertices, edges, faces,

**(2)** the multisets of multiplicities and the multisets of perimeters of faces of $\Gamma_A$ and $\Gamma_B$ coincide,

**(3)** either both or none of $\Gamma_A$ and $\Gamma_B$ are bipartite.

# Affine Type $\mathbb{C}$

Let $R^{\Lambda}(\beta)$ be the cyclotomic quiver Hecke algebra of type $C_{\ell}^{(1)}$, where

$$\Lambda = \Lambda_0 + 2\Lambda_1, \quad \beta = \alpha_0 + \alpha_1.$$

### Proposition (Ariki-Hudak-Song-W., 2024)

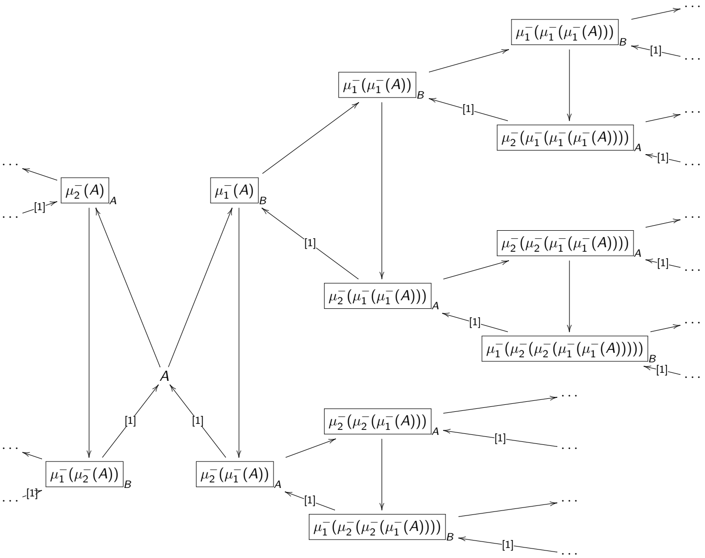
In this case, $R^{\Lambda}(\beta)$ is tame and it is Morita equivalent to the bound quiver algebra $A$ with

$$\begin{tikzcd} \circ \arrow[loop left, "\alpha"] \arrow[r, shift left, "\mu"] & \circ \arrow[l, shift left, "\nu"] \arrow[loop right, "\beta"] \end{tikzcd}$$

bounded by $\alpha^2 = 0, \beta^2 = \nu\mu, \alpha\mu = \mu\beta, \beta\nu = \nu\alpha$.

This is not a Brauer graph algebra!

# Tilting quiver of $A$



Recall that

$$Q: \begin{tikzcd} \circ \arrow[loop left, "\alpha"] \arrow[r, shift left, "\mu"] & \circ \arrow[l, shift left, "\nu"] \arrow[loop right, "\beta"] \end{tikzcd},$$

and define

- $A := KQ/\langle \alpha^2, \beta^2 - \nu\mu, \alpha\mu - \mu\beta, \beta\nu - \nu\alpha \rangle$.
- $B := KQ/\langle \alpha^2 - \mu\nu, \beta^2 - \nu\mu, \alpha\mu - \mu\beta, \beta\nu - \nu\alpha, \mu\nu\mu, \nu\mu\nu \rangle$.

**Proposition (Ariki-Hudak-Song-W., 2024)**

If $C$ is derived equivalent to $A$, then $C$ is isomorphic to $A$ or $B$.

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# Thank you! Any questions?