

## A COMPLETE LIST OF $g$ -VECTORS FOR $\mathcal{H}_4$

We recall that  $\mathcal{H}_4 = \mathbb{F}Q/I$  is given by

$$Q : \begin{array}{ccccc} & & \alpha_1 & & \alpha_3 \\ & & \longrightarrow & & \longrightarrow \\ 1 & \longleftarrow & 2 & \longleftarrow & 4 \\ & \beta_1 & \uparrow & \beta_3 & \\ & & \alpha_2 & \downarrow & \beta_2 \\ & & & & 3 \end{array}$$

with

$$I : \left\langle \begin{array}{l} \alpha_1\beta_1, \alpha_1\beta_2, \alpha_2\beta_1, \alpha_2\beta_2, \alpha_1\alpha_3, \\ \beta_3\beta_1, \alpha_3\beta_3 - \beta_1\alpha_1 - \beta_2\alpha_2 \end{array} \right\rangle.$$

Then, the complete list of  $g$ -vectors for  $\mathcal{H}_4$  is as follows.

$$(1) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$(2) \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$(3) \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$(4) \begin{pmatrix} 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$(5) \begin{pmatrix} 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$(6) \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$(7) \begin{pmatrix} 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$(8) \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$(9) \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}$$

$$(10) \begin{pmatrix} 0 & 0 & -1 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$(11) \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$(12) \begin{pmatrix} 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & -1 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$(13) \begin{pmatrix} 1 & -1 & 0 & 1 \\ 1 & -1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$(14) \begin{pmatrix} 0 & -1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$(15) \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$(16) \begin{pmatrix} 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & -1 & -1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$(17) \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}$$

$$(18) \begin{pmatrix} 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$(19) \begin{pmatrix} 0 & 0 & 1 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$(20) \begin{pmatrix} 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \end{pmatrix}$$

$$(21) \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & -1 \\ 0 & 1 & 0 & -1 \end{pmatrix}$$

$$(22) \begin{pmatrix} 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$(23) \begin{pmatrix} 1 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$(24) \begin{pmatrix} -1 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$(25) \begin{pmatrix} -1 & 1 & -1 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$(26) \begin{pmatrix} 1 & -1 & 0 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & -1 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$(27) \begin{pmatrix} 0 & -1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & -1 & 1 & 0 \end{pmatrix}$$

$$(28) \begin{pmatrix} 0 & -1 & 1 & 1 \\ 1 & -1 & 0 & 1 \\ 1 & -1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$(29) \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$(30) \begin{pmatrix} -1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$(31) \begin{pmatrix} 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 1 & 0 & -1 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$(32) \begin{pmatrix} 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$(33) \begin{pmatrix} 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$(34) \begin{pmatrix} -1 & 2 & -1 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & -1 & -1 \end{pmatrix}$$

$$(35) \begin{pmatrix} -1 & 2 & -1 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}$$

$$(36) \begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$(37) \begin{pmatrix} -1 & 1 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$(38) \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

$$(39) \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ -1 & 1 & 0 & -1 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$(40) \begin{pmatrix} -1 & 2 & -1 & -1 \\ -1 & 1 & 0 & 0 \\ -1 & 1 & 0 & -1 \\ 0 & 1 & 0 & -1 \end{pmatrix}$$

$$(41) \begin{pmatrix} 1 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$(42) \begin{pmatrix} -1 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$(43) \begin{pmatrix} 0 & -1 & 0 & 1 \\ 1 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$(44) \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \\ -1 & 1 & -1 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$(45) \begin{pmatrix} -1 & 2 & -1 & -1 \\ -1 & 1 & -1 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \end{pmatrix}$$

$$(46) \begin{pmatrix} 1 & -1 & 0 & 1 \\ 1 & -1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix}$$

$$(47) \begin{pmatrix} 0 & -1 & 1 & 1 \\ 1 & -1 & 0 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & -1 & 1 & 0 \end{pmatrix}$$

$$(48) \begin{pmatrix} 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 1 & -1 & 1 & 0 \end{pmatrix}$$

$$(49) \begin{pmatrix} 0 & -1 & 0 & 1 \\ 0 & -1 & 1 & 1 \\ 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$(50) \begin{pmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & 1 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$(51) \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$(52) \begin{pmatrix} -1 & 1 & -1 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 1 & 0 & -1 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$(53) \begin{pmatrix} 0 & 1 & -1 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$(54) \begin{pmatrix} -1 & 1 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$(55) \begin{pmatrix} -1 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$(56) \begin{pmatrix} -1 & 2 & -1 & -1 \\ -1 & 1 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & -1 & -1 \end{pmatrix}$$

$$(57) \begin{pmatrix} -1 & 1 & -1 & 0 \\ -1 & 2 & -1 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & -1 \end{pmatrix}$$

$$(58) \begin{pmatrix} 1 & -1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$(59) \begin{pmatrix} 0 & -1 & 1 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$(60) \begin{pmatrix} 0 & 0 & 1 & -1 \\ -1 & 1 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$(61) \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$



$$(62) \begin{pmatrix} -1 & 1 & -1 & 0 \\ -1 & 2 & -1 & -1 \\ -1 & 1 & 0 & 0 \\ -1 & 1 & 0 & -1 \end{pmatrix}$$

$$(63) \begin{pmatrix} 1 & -1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$(64) \begin{pmatrix} 0 & -1 & 0 & 1 \\ 1 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$(65) \begin{pmatrix} 0 & 0 & -1 & 1 \\ -1 & 1 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$(66) \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & -1 & 0 & 1 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$(67) \begin{pmatrix} 1 & -2 & 1 & 1 \\ 1 & -1 & 0 & 1 \\ 1 & -1 & 1 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix}$$

$$(68) \begin{pmatrix} 1 & -2 & 1 & 1 \\ 0 & -1 & 1 & 1 \\ 1 & -1 & 0 & 1 \\ 1 & -1 & 1 & 0 \end{pmatrix}$$

$$(69) \begin{pmatrix} 1 & -2 & 1 & 1 \\ 0 & -1 & 1 & 1 \\ 0 & -1 & 1 & 0 \\ 1 & -1 & 1 & 0 \end{pmatrix}$$

$$(70) \begin{pmatrix} 1 & -2 & 1 & 1 \\ 0 & -1 & 0 & 1 \\ 0 & -1 & 1 & 1 \\ 1 & -1 & 0 & 1 \end{pmatrix}$$

$$(71) \begin{pmatrix} 0 & -1 & 0 & 1 \\ 0 & -1 & 1 & 1 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$(72) \begin{pmatrix} -1 & 1 & -1 & 0 \\ -1 & 1 & 0 & -1 \\ -1 & 1 & -1 & -1 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$(73) \begin{pmatrix} -1 & 1 & -1 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$(74) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$(75) \begin{pmatrix} -1 & 1 & 0 & -1 \\ -1 & 1 & -1 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$(76) \begin{pmatrix} -1 & 1 & -1 & 0 \\ -1 & 1 & -1 & -1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$(77) \begin{pmatrix} -1 & 1 & -1 & 0 \\ -1 & 2 & -1 & -1 \\ -1 & 1 & 0 & -1 \\ 0 & 1 & -1 & -1 \end{pmatrix}$$

$$(78) \begin{pmatrix} 0 & -1 & 1 & 0 \\ 1 & -1 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$(79) \begin{pmatrix} 0 & 0 & 1 & -1 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$(80) \begin{pmatrix} -1 & 1 & 0 & -1 \\ -1 & 1 & -1 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$(81) \begin{pmatrix} 0 & -1 & 0 & 1 \\ 1 & -1 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$(82) \begin{pmatrix} 0 & 0 & -1 & 1 \\ 0 & -1 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$(83) \begin{pmatrix} -1 & 1 & -1 & 0 \\ -1 & 1 & -1 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$(84) \begin{pmatrix} 1 & -2 & 1 & 1 \\ 0 & -1 & 1 & 0 \\ 1 & -1 & 1 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix}$$

$$(85) \begin{pmatrix} 0 & -1 & 0 & 1 \\ 1 & -2 & 1 & 1 \\ 1 & -1 & 0 & 1 \\ 1 & -1 & 0 & 0 \end{pmatrix}$$

$$(86) \begin{pmatrix} 0 & -1 & 0 & 1 \\ 1 & -2 & 1 & 1 \\ 0 & -1 & 1 & 1 \\ 0 & -1 & 1 & 0 \end{pmatrix}$$

$$(87) \begin{pmatrix} 0 & -1 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

$$(88) \begin{pmatrix} -1 & 1 & -1 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$(89) \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$(90) \begin{pmatrix} -1 & 1 & -1 & 0 \\ -1 & 1 & 0 & -1 \\ -1 & 1 & -1 & -1 \\ 0 & 1 & -1 & -1 \end{pmatrix}$$

$$(91) \begin{pmatrix} 0 & -1 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$(92) \begin{pmatrix} 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$(93) \begin{pmatrix} 0 & -1 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$(94) \begin{pmatrix} 0 & -1 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$(95) \begin{pmatrix} 0 & -1 & 0 & 1 \\ 1 & -2 & 1 & 1 \\ 0 & -1 & 1 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix}$$

$$(96) \begin{pmatrix} 0 & -1 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$